SI Manual

What is Supplemental Instruction (SI)?

- Supplemental Instruction (SI) consists of group study sessions set up for specific courses.
- Instructors who elect to participate in SI are paired with a student of their choosing, known as an SI Leader. The SI Leader must have passed the course to which they are assigned successfully (with an A or a B) or placed at a higher level.
- Each group tutoring session is led by the SI Leader outside of class.
- The SI Leader attends the course all semester with you, the instructor, in order to provide relevant group discussions of course concepts with an emphasis on study skills and learning strategies.

Responsibilities of the SI Instructor

- Meet with SI Leader 30 minutes each week of the semester to share teaching ideas for the SI sessions.
- Allow SI Leader to make announcements at start of each lecture, roughly 5 minutes, to promote SI.
- Allow SI Leader to survey class at two points in the semester.
  - Beginning of semester for SI session scheduling.
  - End of semester for input on effectiveness.
- Recommend students as SI Leaders on SI Instructor Application, which is usually sent around the 13th week of the semester.

For more information on SI, visit the Bakersfield College SI Website, www.bakersfieldcollege.edu/si, or contact Eileen Pierce, whose contact information is listed below.

Eileen Pierce Office: CSS-193 Phone: 661-395-4202 Email: epierce@bakersfieldcollege.edu

Table of Contents for Remainder of SI Manual

Supplemental Instruction (SI) Instructor Application

Instructor: Department:

Today's Date:

Class(es) needing SI support (include all sections of same course)**:**

- Course name: _____ CRN # _____
- Course name: _____ CRN # _____
- Course name: _____ CRN #_____

*Add more if you need to…

Please answer the following questions:

1. Why do you think SI is an effective support system for your course(s)?

2. What is the expected enrollment for each section?

3. Your duties as an SI instructor include: **1)** meeting with your SI Leader weekly for at least 30 min. to discuss course content and progress of SI sessions, **2)** consistently promoting the SI program to your students verbally and supporting your SI Leader's efforts, and **3)** attending a 60-minute faculty orientation prior to the start of the term during FLEX week (not mandatory but strongly, strongly encouraged so that you'll have a full grasp of the goals of the program).

   Do you have sufficient time to fulfill the obligations mentioned above?

4. Who do you recommend as your SI Leader(s)?: PLEASE RECOMMEND 2-4 STUDENTS if possible!
   *NEW (Please discuss SI with your recommendees ahead of time)

NAME:	ID#:	BC e-mail address (up to @ sign):

*RETURNING from previous semester:

NAME:	ID#:	BC e-mail address (up to @ sign):

Note: Returning SI Leaders have priority in hiring, and then I will do my best to accommodate requests for new SI Leaders given availability of instructors, scheduling compatibility, and budget constraints.

Please acknowledge: ☐ I have read all of the above and am aware of my responsibilities as an SI Instructor.

Please return this application to Eileen Pierce, ext. 4202:
***via e-mail: epierce@bakersfieldcollege.edu**
***campus mail/in person: SS-193**

Supplemental Instruction (SI) Fact Sheet for Prospective SI Leaders For Summer/Fall 2016

The Essential Elements of SI

*Targets subjects, not students (i.e., not remedial)
*Sessions facilitated by peer (you!)
*You act as a model student
*Combines content and study skills
*You attend ALL students' classes
*You receive regular training
1st training for Summer—will be announced via BC e-mail

Role of the Leader

*Facilitate 2-3 hrs/wk of group study sessions
*Clearly state expectations to students
*Establish rapport & build relationships
*Empower students to help themselves
*Attend class every week—take class notes & identify student needs
*Meet with instructor weekly-30 min.
*Promote SI sessions in class DAILY
*Remind students to sign in for sessions

What Your Weekly Job Schedule Would Be: (for each section you do SI for!)

*You are paid $10/hr. for the following:
--Attendance at all class meetings (usually 3-4 hrs/wk, 12 hrs/wk in Summer)
--Weekly SI sessions: 2-3 @ 1 hr each (per section)
--Instructor meeting: 30 min per week
--Prep time: 1.5 hrs per week per section
--Training sessions (3-4 per sem)

Ready to apply?—Contact SI Program Manager, Mrs. Eileen Pierce: 395-4202, SS-193, or epierce@bakersfieldcollege.edu

Additional notes:
*Must have a specific instructor/class in mind for which you wish to provide SI and be recommended by that instructor (on the SI Instructor Application sent out April 11)
*Must have passed that specific course already with an A or B OR tested at a higher level on Placement Test
*Must carry a minimum of 6 units for Spring/Fall terms and ½ unit during Summer session!

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: _____ CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?

_____

_____

_____

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up—	
Work-out--	
Cool-down--	

See backside ⇨

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

3. Attach copies of any handouts you used.

Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
(Habits of Mind) What does it mean to PERSIST?
How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

Policy: Turn in 1 completely filled out Session Plan per week every Monday by 5 PM to avoid a Missing Paperwork warning.

Narrative of SI Recruitment

In this section, we present the reflections of a BC math professor who elected to employ a Supplemental Instruction leader in his Fall 2016 Math B70 course. The narrative is written in the first person voice to underscore the professor's perspective as he navigated the processes of identifying and recruiting his first SI leader.

Identifying a strong candidate. My main criteria for finding a student to be my SI leader were to find someone who was compatible with my teaching style, and who shared a similar philosophy as to what math is and how people learn math. I believe that students learn by doing math, talking about math, and by explaining their own, as well as others', mathematical thinking. Thus, I emphasize problem solving in my classrooms, and I query students concerning their thinking as they work towards a solution strategy. Therefore, I was looking for a student who demonstrated strong conceptual knowledge, worked well in a group setting, and to whom other students gravitated toward and respected. I wanted an SI leader who could be another pair of eyes for me in the classroom, who could work with me to identify other students' mathematical weaknesses, and who could develop supplemental instruction lessons to assist those students. Essentially, I wanted an SI leader who had: (a) strong content knowledge, (b) excellent communication skills, and (c) exceptional leadership qualities. In the following paragraphs, I provide more insight as to what I was looking for in the aforementioned set of desirable traits.

Content knowledge. As for content knowledge, I wanted someone who understood underlying mathematical concepts and who could also make the links between specific concepts and those procedures that incorporated the concepts. I wanted someone who could proficiently manipulate mathematical symbols, yet, also explain why the procedural steps worked, and, when appropriate, explain the connections between symbols and their respective referents in a story problem. Furthermore, I was looking for someone who demonstrated the ability to "unpack" a problem (especially a story problem), examine multiple entry points to a problem, and then explore different solution paths and strategies. I wanted someone who was not married to a specific procedure for a specific problem, and who demonstrated flexibility in her/his mathematical thinking. This was important to me because I wanted an SI leader who could view a problem from many angles and understand that there is always more than one way to solve a problem. I wanted an SI leader that could help an individual based upon that person's particular way of thinking and not force that person to use a strategy that may be too sophisticated for that person at that moment in time.

Communication skills. Because I wanted an SI leader who could successfully interact with students both in and out of the classroom, I needed to find an individual who could clearly explain her/his thinking using language that was understandable to other students. I wanted to find someone with a mathematical voice different from my own, that is, someone who could explain concepts or procedures from a different perspective, a perspective that might make better sense to other students. I was looking for someone who was able to provide detailed explanations to questions I asked in class, but in a manner different than I might, and who was not afraid to ask questions if a concept was not immediately understood. I wanted someone who took ownership of her/his own learning, but was also concerned about the other students in the class. This concern for other students segues to the final quality I was looking for – leadership.

Leadership skills. I use a lot of group activities in my classroom so I was looking for a student who took control of her/his group without being overbearing. I wanted someone who listened to other students and valued different input and ideas but who, in turn, offered help when necessary and responded to group members in a respectful manner. An ideal SI candidate would also offer help to other students outside of the group if asked and help both known acquaintances as well as others. The SI candidate I finally decided upon went above and beyond my expectations in this regard. As a student in Spring 2016, he not only took a leadership role when in a group setting, he also organized outside study groups, and for the final exam, he organized a study session at his local church and invited the entire class to attend the session. The student that I finally identified met all of my criteria, and more. Now I just had to convince him to work with me as an SI leader.

Recruiting the candidate. My initial conversation with the student was basically to find out if he had ever heard of Supplemental Instruction at BC, and if he knew what an SI leader was and what one did. In this case, the candidate was not aware of SI, what it entailed, and what role he might play as an SI leader in my classroom. (Since these details are found in other sections of this manual, I won't go into them, rather I will explain what steps I followed once I had gauged that he was interested.)

Recruitment pitch. First, I sent him a copy of the SI Fact Sheet, which provided to him an overview of the job, and then I met with him to discuss how we might form a strong partnership in the fall. To me that was a key component of my recruitment pitch – the idea that we were to be partners in the experience. I explained that he would be another set of eyes and ears in the classroom, and that I had watched him enough to fully trust his judgment when working with other students. In addition, I explained that I would help him to decide what topics that he should emphasize and cover in the Supplemental Instruction weekly sessions. I told him that together we would discuss what concepts the class as a whole, or specific individuals, might be struggling with, and then we would develop an intervention strategy. I did emphasize, however, that he would have autonomy during the SI sessions, and that I did not expect him to, nor did I want him to mimic what I did in the classroom. In closing, he agreed, and now I am very excited to pilot Supplemental Instruction during the upcoming semester, and beyond.

Summary. To encapsulate the process of identifying and recruiting an SI Leader, I offer the following set of bulleted points.

SI Leader Characteristics

1. Content knowledge.
   (a) Understands concepts and procedures
2. Communication skills
   (a) Clearly explains own thinking
   (b) Offers alternative perspectives
3. Leadership skills
   (a) Takes control of a group setting
   (b) Offers help to classmates

Recruitment

1. Have a conversation to explain Supplemental Instruction
2. Gauge the student's interest
3. Send the SI Fact Sheet
4. Emphasize that SI is a partnership

In this section, we present the insights from a few BC Math and Science professors who have used SI for multiple semesters and who were willing to share their comments. Each has continued with SI. While content knowledge, subject matter competence, communication skills, and leadership were all identified as of utmost importance, they are left out here. The intent of this section is to share insights gained by having used Supplemental Instruction for multiple semesters. The goal is to help new teams of faculty and SI Leaders experience confidence and success as soon as possible!

The Successful SI Leader

- Exhibits respect for students, SI Faculty Member, and self. Shares a positive outlook on life.
- Establishes and maintains boundaries
  - Students seeking help may
    - want to spend time blaming the professor for not teaching well
    - have personal problems and want to lean on the SI Leader for personal support in difficult times
    - accuse the SI Leader of not helping them get homework done
  - SI Leaders who have met with the SI Faculty Member and who have planned well are able to
    - redirect attention to the subject matter to be learned
    - deflect inappropriate comments and behavior
    - avoid taking negative comments personally
    - avoid battling students
    - be warm, supporting, and caring, while focusing on the plan for the day
    - encourage and present as planned explaining that SI does not replace doing homework
    - share concerns with SI Faculty Member and with the director of SI on campus so that particular students may be directed to seek help from established campus resources if deemed necessary
    - share information about material that students are finding difficult so that the faculty member may adjust lectures or help SI Leader design targeted SI Sessions
- Explains and shares strategies for studying and learning including
  - how to take, review, and reorganize class notes
  - methods to help learn from the book (definitions, checking answers, chapter reviews)
  - awareness of metacognitive processes
    - take notes on thinking while doing math
    - write down thoughts if in a moment of confusion or during a successful breakthrough (both may be useful in an office hour)

  - Habits of asking himself/herself questions while working through a problem
    * Does the step I am doing make sense?
    * Is there a better way to write the answer?
    * Read and reread directions associated with any problem or question. Once completed, reread the directions again to ensure successful completion.

      Check answers when possible.

- Manages time well so that SI expectations are met while succeeding in his/her own classes as he/she
  - attends every class session
  - attends all SI instruction
  - meets with SI Faculty regularly
- Cooperates with the expectation/desire of the individual SI Faculty Member with respect to
  - following methods outlined in class vs. showing new methods not covered in class
  - using "tricks" to do problems
  - using content strategies that are appropriate to the current class level and avoiding introducing procedures that are taught in later content classes
- Exhibits correct use of mathematical terminology and logic

Strategies Used by SI Faculty that Encourage Student Participation

- Invite SI Leader to solve student homework problems at the board for a few minutes once a week
- Invite SI Leader to describe the plan for the upcoming SI Session
- Offer 2 points on the next in-class exam for turning in a summary of the SI Session
- Offer extra credit for participation (One math teacher offered up to 1% of the total grade for participation and had approximately 60% of her students participate. Participation was only 10% in another class with another faculty member who did not offer extra credit.)
- Offer extra credit for SI as one choice among others including using on-campus tutoring, working with STEM tutors so that students whose schedules do not work do not feel penalized
- Ask SI Leaders to sign a form that students must turn into the faculty member to get the extra credit for participating

Benefits of SI Participation

- SI Leader
  - Receives training from the SI Office and other experienced SI Leaders and the SI Faculty Member
  - Increases in confidence and ability in the subject matter
  - Experiences the joy of teaching and serving others
    Is introduced to the career of being a community college professor
- Participating Students
  - increase in confidence in content area
  - show improvement on tests
  - Earn higher grades in class (on average)
  - Build lasting study skills both as individuals and as members of study groups that can be used in future classes
- SI Faculty Member
  - Experiences the satisfaction of mentoring a bright college student
    Enjoys increased preparation of students in class
- Non-participating students
  - Have more prepared students around them
  - Can be helped by classmates informally before and after class

Note: A class with an effective SI Faculty Member and SI Leader benefits from the establishment of a collegial learning environment where students, SI Leader, and SI Faculty Member work as a team to help everyone succeed. Measures of effectiveness of SI may include the number of students who participate at least once, the total number of students attending, or the average attendance per session. But the benefits of participation do not depend solely on these numbers. Even if numbers are low, the SI Leader and faculty member are establishing a mentoring relationship and the student is given an entry level position in the profession of college teaching. One retired math faculty member at a college different from BC shared that a large percentage of her SI leaders over the years have gone on to become college professors.

Perspectives from experienced SI Leaders and Faculty Members that help guide thinking, preparation, and speaking about SI to others.

SI IS NOT

- tutoring
- counseling
- social time
- designed exclusively for weak or underprepared students

SI IS DESIGNED

- to improve student thinking and content area skills
- to have student feel safe while studying the content
- to help students make lasting connections and friendships as they study and work together
- to build student confidence and strength
- to enable students to experience personal growth
- for all students!

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: _____Math B50_____ CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
____The simplification of expressions containing signed numbers using the order of operations._____
_____
_____

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up-- See if anyone can work out this	Give students time to attempt the warm-up question,
problem from last session: $-12 + (-7) - 15$.	then discuss any responses to the warm-question as a
	group.
Work-out--	Work through Problems 1 and 2 for the students, then
1. $4 \cdot 3^2$	answer any questions that arise. Have the students pair
2. $2 + 9 \cdot 5 - 16$	up to work through Problems 3 through 8 with one
3. $\lvert 12 - 21 \rvert \cdot (-2) \div (-3)$	another. After working in pairs, invite the entire group to
4. $14 \div 2 \cdot 7 + 1^3$	work together, fielding questions as they arise.
5. $(7 - 3^2)^2$	
6. $8(5 - 2) + (-2)^2 - 2^2$	After the group has worked together, discuss study
7. $6[(3 + (-7) \cdot 2 + (2 + 10)]$	strategies such as note taking, flash cards, and mnemonic
8. $\frac{15 - (3)(-4)}{2[-10 \div 5 \cdot (-2)] + 1}$	devices. Caution students on the "hurried" use of devices
	like PEMDAS. For example, in Problem 4 note that
Introduce and play the board game Equate, if time	the operation of division is performed before the
allows.	operation of multiplication even though "M" appears
Cool-down-- Try to give a clear, concise explanation	before "D" in PEMDAS.
of the material covered in our session today.	
	If time allows, play Equate, finishing with the cool-down.

See backside ⟹

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

3. Attach copies of any handouts you used.

Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
(Habits of Mind) What does it mean to PERSIST?
How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

Policy: Turn in 1 completely filled out Session Plan per week every Monday by 5 PM to avoid a Missing Paperwork warning.

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: _____Math B60_____ CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
_____ To study the connection between rate and slope including examples with graphs. _____
_____
_____

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up-- How would you describe the slope of a	Ask for responses to the warm-up question and
(nonvertical) line?	discuss any responses as a group.
	Have students get into small groups and begin working
Work-out-- Please see attached handout.	on the handout. Walk around to each group, pausing at
	each to listen to ideas, offer suggestions, or take
	questions.
	Assuming enough groups have been formed, ask one
	member from each group to work through a problem on
	the board. Adapt, as needed, given the number of groups
	until all problems up to, but not including, Problem 6
	have been explained. Have students work on Problem 6
	individually before revealing the solution and discussing it
	as a group.
Cool-down-- Of the concepts mentioned today,	Share any tips that come to mind about developing
which do you feel still requires the most practice to	and remembering approaches for common problems
master?	involving slope. Finish with the cool-down.

See backside ⇒

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

3. Attach copies of any handouts you used.

Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
(Habits of Mind) What does it mean to PERSIST?
How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

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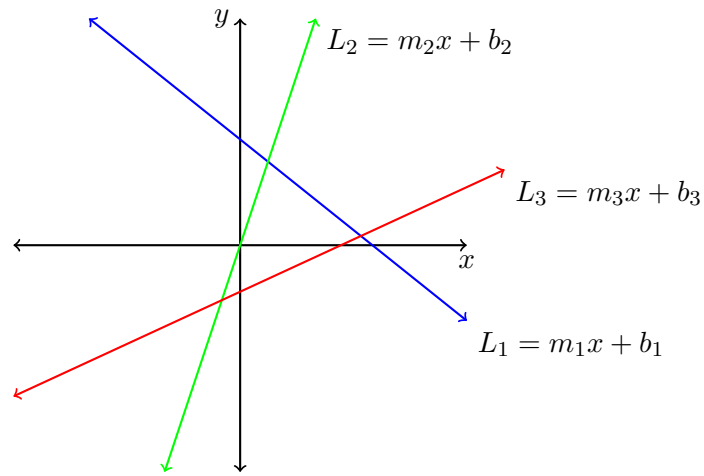
Handout for SI Session Plan - Math B60

1. Consider the lines L_1, L_2, and L_3.

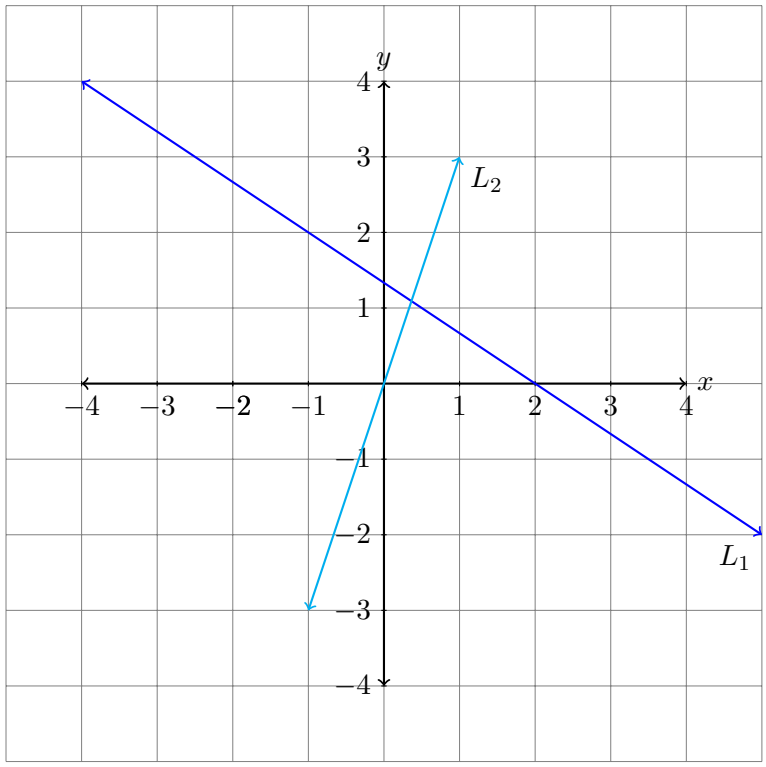
   (a) List the slopes in increasing order.

   (b) List the y-intercepts in increasing order.



2. Use point-slope form to find an equation of the line that contains $(-2, -5)$ and that is perpendicular to $x + 2y = 5$.

3. Find the slope of each line.



4. Use point-slope form to find an equation of the line that passes through $(-1, -3)$ and $(-3, -9)$. Write your final answer in slope-intercept form.

5. Jane decides to take her motorcycle to a nearby racetrack for a track day. As she approaches the start-finish line of the track, she notices the timer on the side of the track reads 4:18, indicating that 4 minutes and 18 seconds have passed since the start of the session. She gets a rolling start, passes the start-finish line at 4:22, and proceeds to do a flying lap. Given that she crossed the start-finish line at the end of her flying lap at 6:37 and that the track is 4.8 kilometers in length, answer the following questions.

   (a) Find her rate of travel, in kilometers per hour.

   (b) Find the rate of travel, in laps per minute.

6. Jerry bought a piece of equipment for his company which cost \$17,000. The value of the piece of equipment over several years is shown below.

Year, x	Value, y
0	\$17,000
1	\$16,000
2	\$15,500
3	\$3,000

(a) Draw a graph of the data. Be sure to label axes with respect to the context of the problem.

(b) What is the slope from year 1 to year 2?

(c) What is the slope from year 2 to year 3?

(d) Are there any data values that seem to be odd or unusual? If so, what do you think might have happened to the piece of equipment to lead to these data values?

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: _____Math B70_____ CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
_____To solve systems of two linear equations in two variables using the substitution method and the elimination method._____

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up-- What does it mean for a system of	Ask for responses to the warm-up questions and
equations to be consistent? What does it mean for a	discuss any responses as a group.
system of equations to be inconsistent?	
	Demonstrate Problem 1 (a) using the substitution
Work-out-- Please see attached handout.	method and Problem 1 (b) using the elimination method.
	Invite the group to discuss the efficiency of the two
	methods. Ask the group to continue with Problems 1 (c)
	through (h) individually. Afterwards, ask students to
	demonstrate at least one problem to another student.
	As a group, discuss any study strategies to help
	remember the differences and similarities of the two
	methods. Allow students to work through Problems 2 and
	3 with a partner. Discuss the solutions as a group,
	emphasizing the importance of parsing through the
	statement of the problem to identify all necessary
Cool-down-- Which problem that you understood	information.
today left you with the most confidence?	
	Finish with the cool-down.

See backside ⇒

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

3. Attach copies of any handouts you used.

Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
(Habits of Mind) What does it mean to PERSIST?
How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

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Handout for SI Session Plan - Math B70

1. Solve.

   (a) $\begin{aligned} 2x + 5y &= 12 \\ y &= 3x - 1 \end{aligned}$

   (b) $\begin{aligned} 4x + 3y &= 4 \\ 2x - 3y &= -16 \end{aligned}$

   (c) $\begin{aligned} 7x - 2y &= 1 \\ x &= 3y + 11 \end{aligned}$

(d) $\begin{array}{c} -5x + 5y = 10 \\ 5x + 2y = 4 \end{array}$

(e) $\begin{array}{c} -2x + 3y = 1 \\ 7x + y = 8 \end{array}$

(f) $\begin{array}{c} 4x - 9y = -2 \\ 2x - 3y = -4 \end{array}$

(g) $\begin{array}{c} 2x + 6y = 9 \\ x = -3y + 14 \end{array}$

(h) $\begin{array}{c} 21x - 12y = 15 \\ 7x - 4y = 5 \end{array}$

2. Jane needs 4 L of a solution that is 20% base, but she only has two solutions available to her at the moment, neither of which is 20% base. One solution is 15% base while the other solution is 35% base. How much of each available solution should Jane mix to yield 4 L of a solution that is 20% base?

3. A wolf leaves its den and travels northwest at 10 km/h. An hour later, the rest of the pack leaves the den and follows the same path the first wolf took, traveling at 15 km/h. How far from the den will they meet?

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: Math B60 or Math B70 CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
Accomplish measuring perimeter, area, and containment of wildfires in the attached handout.

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up-- Discuss current events of interest. Point out	How are you all doing? What are some interesting
math applications if natural to do so. Remind students of	news items this week? Do you remember any fires in
any recent wildfires (or older wildfires) and give them	California during Summer 2016? If so, what did reporters
time to see what they remember.	tell us about the fires, and what did that mean? Did
Work-out-- Please see attached handout. Make a copy	anyone have friends or family involved? If so, what was
for each student, but give them only one page at a time	their involvement? (e.g. victim, volunteer, firefighter)
to work on individually, in pairs, or as a group. As a	
suggestion, cover the first page of the handout slowly	Begin the work-out with the attached handout. (Please
and in detail before distributing the second page of the	see the notes to the left.)
handout. Check each student's answer to "Fire X" before	
having them attempt "Fire Y" and "Fire Z."	
Cool-down-- The cool-down may be the fourth page of	Finish with the cool-down.
the handout or a wrap-up discussion after the fourth	
page. Review the main idea found under the table on the	
fourth page and the importance of three measures:	
area burned, perimeter, and containment.	

See backside ⇒

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

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Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
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How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

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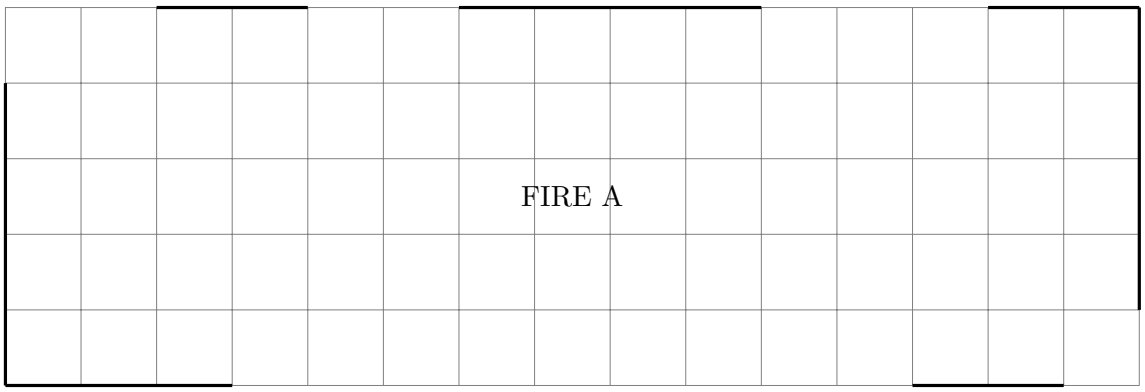
Measuring Area, Perimeter, and Containment of Wildfires

1. Suppose there is a wildfire that is burning in the shape of the rectangle drawn below with 1 cm = 1 mi used as the scale of the drawing. Examine the given rectangle and answer the questions that follow.



   (a) What is the area of Fire A?

   [The formula we use to find the area of a rectangle is $Area = Length \times Width$.]

   (b) What is the perimeter of Fire A?

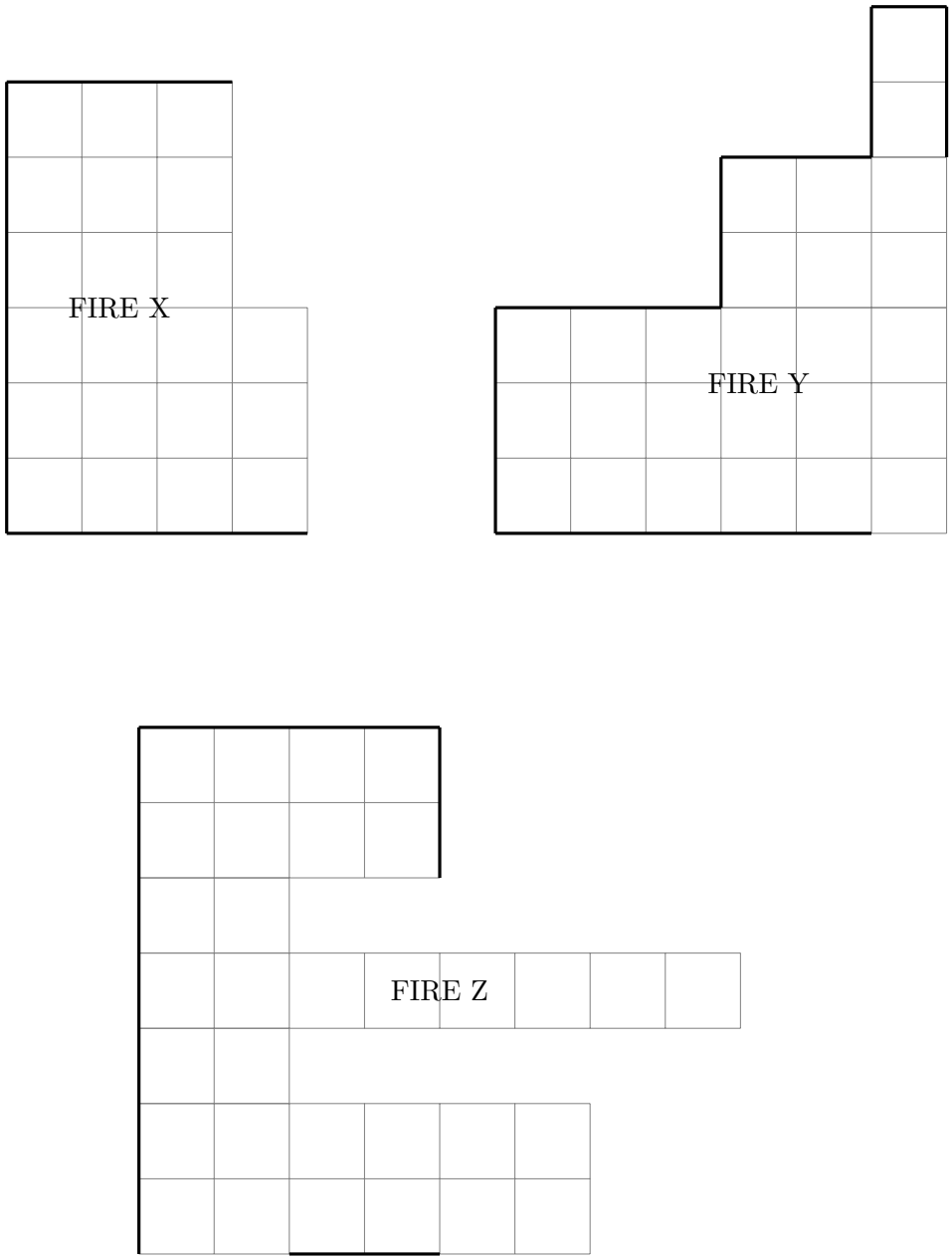
   [The formula we use to find the perimeter of a rectangle is $Perimeter = 2 \times Length + 2 \times Width$.]

   (c) In the figure above, part of the outside boundary is dark and part is light. If the dark represents the portion of the outer edge of the fire that has been contained, we can measure the percentage of the whole perimeter that is contained.

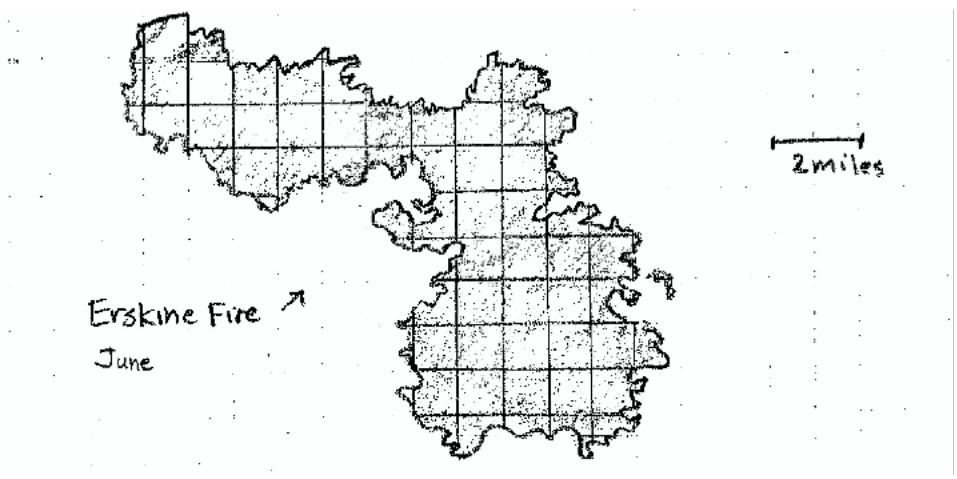
   - Step 1: Find the length of the contained outer edge of Fire A.
   - Step 2: Make a fraction with the contained portion of the outer edge in the numerator and the entire perimeter in the denominator.
   - Step 3: Find the equivalent decimal and percent representation of the fraction found in Step 2. The percent you have found is the percentage of containment of Fire A.
   - Step 4: Fill in the blanks to complete each sentence. Fire A has burned _____ square miles. Fire A is _____% contained.

Note: Firefighters say that a fire is 100% contained when the ability of a fire to spread on the ground has been stopped along its whole perimeter. The fire may still be burning in the interior region of the area of burn. And, embers may fly through the air and cause new burn in nearby areas, so there is still more to do once the fire is 100% contained. Also, if structures are involved, there are potential chemical, gas, and electrical dangers involved in reentering a burned region.

2. A figure is provided below for each of three pretend fires: Fire X, Fire Y, and Fire Z. For each figure, the burned area is the interior of the figure. The perimeter is the outside boundary. Dark outside boundary edges represent containment while light borders represent no containment of the fire's progress. Assume that each 1 centimeter represents 1 mile. Each square of the grid in the figures represents 1 square mile. For each figure, calculate the area of burn, the total perimeter of the fire, and the percentage of containment of the fire.



3. The Erskine Fire burned for more than a week in the summer of 2016. After five days, approximately 45,000 acres had burned, and the map of the fire looked like the figure below with 1 mile represented by 0.25 inch. Note that the area of burn is described in acres. Each square mile is equivalent to 640 acres. So, two square miles would be 1280 acres, and ten square miles would be 6400 acres. In the figure below, each square on the grid represents 1 square mile. Note that the area and perimeter of real fires are more difficult to calculate because they have shapes that are not always rectangles.



(a) Using the grid provided, estimate the burned area of the Erskine Fire as represented in the figure presented above using square miles.

(b) One square mile is equal in area to 640 acres. Rewrite your answer in part (a) using acres as your unit of measure. How many acres are represented in the figure provided?

(c) Compare your answer in part (b) to the reported number of acres of 45,000 acres. Does the figure make sense for this amount?

(d) Discuss how difficult it would be to find the perimeter of the fire with the figure provided above.

4. The table below has information about the Erskine Fire during many of the first 8 days it was burning. The information has come from KGET Channel 17 TV News Reports in Bakersfield, CA and the website KernGoldenEmpire.com. It is interesting to read all of the items in the first column. Each label is an important measurement of a fire. Fires are not just measured by area or containment. They are also measured by the length of time, expected end of expansion (expected 100% containment), destruction caused (number of structures destroyed, number of structures damaged, number of deaths caused), number of people involved in fighting the fire, and the cost of the firefighting efforts.

Data from KGET Channel 17 TV News Reports on Fire in Erskine, CA

Day	Day 1: 6/23/16	Day 3: 6/25/16	Day 5: 6/27/16	Day 7: 6/29/16	Day 8: 6/30/16
Expected 100% Containment	Not Reported	Not Reported	Midnight on 6/30/16	Midnight on 6/30/16	7/5/16
Acres Burned	5000	35,711	45,388	46,679	46,684
Containment	0%	0%	40%	60%	70%
Structures Destroyed	80	150 or More	Not Reported	257	257
Structures Damaged	Not Reported	75	Not Reported	17	17
Number of Personnel	350	1139	2079	1743	Not Reported
Firefighting Cost	Not Reported	$3 million	Not Reported	$15.8 million	$17.5 million

Several different measurements of the fire are reported each day.
The area of the fire is measured in acres.

Discuss the different measurements found in the table. Measurements are used to describe, record, and communicate important characteristics about events, people, or things to enable understanding, need, and comparison to similar events.

5. While the Erskine Fire burned, two other fires broke out near Bakersfield. The Stallion Springs Fire in Tehachapi burned 18 acres and was 100% contained at the end of its first day. The Deer Fire near Arvin and Bear Valley Springs had burned 35 acres and was 0% contained at the end of its first day.

   (a) Discuss what each of these measurements means and how these data compare to that of the Erskine Fire on its first day.

   (b) Discuss the potential for further damage for each of these two fires from the perspective of the first day.

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: Math B50, B60, B70, B1A, B4A CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
Learn the essential ideas involved in finding sums of all whole numbers from 1 to 100 as Gauss might have done as a child. Students will also try to generalize to find the sum of all whole numbers from 1 to n, where n is a whole number greater than 1.

Before you start, don't forget:

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2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up—	Let's have you add up the integers from 1 to 20, but
Students add some sums of numbers:	first warm up with these sums:
1 + 2 =	1 + 2 =
1 + 2 + 3 =	1 + 2 + 3 =
1 + 2 + 3 + 4 =	1 + 2 + 3 + 4 =
1 + 2 + 3 + 4 + 5 = .	1 + 2 + 3 + 4 + 5 = .
Work-out--	
Calculate 1 + 2 + 3 + ⋯ + 18 + 19 + 20.	← Try this now. Using any method you can think of, find
	the sum. Have any of you heard of Johann Carl Friedrich
Introduce Gauss.	Gauss?
Please see attached handout	
for the rest of the SI Session Plan.	
	Finish with the cool-down. Write some notes to help you
	remember how to add the whole numbers from 1-100 as
Cool-down-- Write down one example from today's	young Gauss might have in the 1780's. Write a formula
work with notes to help remember the process.	for 1 + 2 + 3 + ⋯ + n also!

See backside ⟹

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

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Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
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Johann Carl Friedrich Gauss and Finding the Sum of the Whole Numbers from 1 to 100

Johann Carl Friedrich Gauss was a German mathematician who contributed much to our understanding of many applications and theories in mathematics and physics. According to a fun story about Gauss as a young child, a teacher gave him the job of adding up all the whole numbers from 1 to 100. It is supposed that the teacher may have wanted to challenge Gauss or keep him busy for a period of time. The teacher may have expected Gauss to do a sequence of 99 sums: $1 + 2 = 3, 3 + 3 = 6, 6 + 4 = 10, \ldots, 4851 + 99 = 4950$, and $4950 + 100 = 5050$. Instead, according to the story, he quickly responded with the correct answer of 5050.

How might Gauss have calculated the correct answer so quickly and been so confident in his answer? There are different explanations that have been suggested based on a single essential idea. The essential idea is that there are 100 numbers or 50 pairs of numbers. If we add the first and last together and work our way into the middle, we get 50 sums of 101. Therefore, the sum will be $50 \cdot 101 = 5050$.

Explanation 1

We want to find $1+2+3+\cdots+98+99+100$. Note that there are 100 numbers and that the sum of the smallest number and the largest number is 101. That is, $1+100 = 101$. Note further that $2 + 99 = 101$ and $3 + 98 = 101$. In fact, there are exactly 50 pairs of numbers whose sum is 101. Why are there 50 pairs? Well, $100/2 = 50$. We now multiply 50 (the number of pairs) and 101 (the sum of each pair) to get 5050.

$$1 + 2 + 3 + 4 + 5 + \cdots + 49 + 50 + 51 + 52 + \cdots + 96 + 97 + 98 + 99 + 100$$

$$= (1 + 100) + (2 + 99) + (3 + 98) + (4 + 97) + (5 + 96) + \cdots + (49 + 52) + (50 + 51)$$

$$= \underbrace{101 + 101 + 101 + \cdots + 101}_{50}$$

$$= 50(101)$$

$$= 5050$$

Conclusion: $1 + 2 + 3 + \cdots + 100 = 5050$.

Explanation 2

Write the numbers 1 to 100 twice, once in increasing order and once in decreasing order. Then, add these two lists of numbers vertically!

$$\begin{array}{ccccccccccccccccccc}
1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & 97 & + & 98 & + & 99 & + & 100 \\
100 & + & 99 & + & 98 & + & 97 & + & \cdots & + & 4 & + & 3 & + & 2 & + & 1 \\
\hline
101 & + & 101 & + & 101 & + & 101 & + & \cdots & + & 101 & + & 101 & + & 101 & + & 101
\end{array}$$

We then have 101 listed 100 times. But this sum is $100(101) = 10,100$. This number is twice the amount we are looking for, so we can calculate $\frac{(100)(101)}{2} = 5050$. (Note that $\frac{(100)(101)}{2}$ is equivalent to $50(101)$, the product found in Explanation 1.)

Explanation 2 Revisited

We want to compute $1 + 2 + 3 + \cdots + 100$.

Let $x = 1 + 2 + 3 + \cdots + 100$.

It is also true that $x = 100 + 99 + 98 + \cdots + 1$.

$$\begin{array}{ccccccccccccccccccccc}
x & = & 1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & 97 & + & 98 & + & 99 & + & 100 \\
x & = & 100 & + & 99 & + & 98 & + & 97 & + & \cdots & + & 4 & + & 3 & + & 2 & + & 1 \\
\hline
2x & = & 101 & + & 101 & + & 101 & + & 101 & + & \cdots & + & 101 & + & 101 & + & 101 & + & 101
\end{array}$$

$2x = 100(101)$ is the result of adding the two equations together vertically. Next, we divide both sides of the resulting equation by 2 to get $x = \frac{(100)(101)}{2} = 5050$. Therefore, we conclude that $1 + 2 + 3 + \cdots + 100 = 5050$.

Practice Problems

1. Find the sum $1 + 2 + 3 + \cdots + 60$.
2. Find the sum of the integers from 1 to 200.
3. Find the sum of the integers from 1 to 1000.
4. Generalize to find the sum of the integers from 1 to n, where n is an integer greater than 1.
5. Find the sum $57 + 58 + 59 + \cdots + 107$.
6. Find the sum $2 + 4 + 8 + \cdots + 600$.
7. Find the sum $1 + 3 + 5 + \cdots + 115$.
8. Make up four problems of your own to share with a classmate.

Note: The answer to Question 4 is $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

Enjoy! Gauss contributed to many aspects of human understanding of mathematics and physics. Look him up to learn more!

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: Math B60 or Math B70 CRN: _____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
To master the definition of a prime number, a composite number, and the use of the Sieve of Eratosthenes to identify all the prime numbers between 1 and 30 and also between 1 and 100.

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up—	Say hello, and ask how everyone is doing today. Begin
1. Definition of a prime number.	with the warm-up. Discuss the definitions in (1.) and (2.)
2. Definition of a composite number.	of the warm-up section. Ask for responses to (3.) and
3. Give an example of (1.) and (2.).	keep a running list of the prime and composite numbers
4. Classify each number in the following list as	that are said. Finally, have the group work on (4.)
prime or composite: 2, 19, 29, 39, 53, 57, 59, 80.	together. Provide any needed corrections, and then move
Work-out--	to the work-out section.
I. Classify the integers from 1 to 30 as prime,	I. Have the whole group brainstorm, using
composite, or neither. (Note that 1 is neither prime nor	anything that comes to mind. (e.g. memory, definitions
composite.)	After discussing the task, ask the students what they
	would do if the task were to classify the integers from 1
II. Introduce and use the Sieve of Eratosthenes	to 500 instead of 1 to 30. Would their strategy change?
using the accompanying handout to classify the integers	II. Use the attached handout to introduce and
from 1 to 30 as prime, composite, or neither.	use the Sieve of Eratosthenes for the "1 to 30" task.
	III. Use the attached handout to use the Sieve of
III. Use the Sieve of Eratosthenes and the	Eratosthenes for the "1 to 100" task.
accompanying handout once again to classify the integers	In the cool-down, in addition to discussing the ideas to
from 1 to 100 as prime, composite, or neither.	the bottom-left, introduce the man Eratosthenes and talk
Cool-down-- Tie together the desire to find prime	about the word "sieve." Discuss the human aspect and as
numbers and discuss how computers can help.	many of the "reflections" in the handout as time allows.

See backside ⟹

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

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How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
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Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
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The Sieve of Eratosthenes

Using the Sieve of Eratosthenes to Find the Prime Numbers Between 1 and 30

Once we decide that we want to find all of the prime numbers between 1 and 30, we write all of the integers between 1 and 30. It does not matter if we write the numbers horizontally or vertically, as long as we write them neatly and in order from least to greatest.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

1	11	21
2	12	22
3	13	23
4	14	24
5	15	25
6	16	26
7	17	27
8	18	28
9	19	29
10	20	30

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

1	6	11	16	21	26
2	7	12	17	22	27
3	8	13	18	23	28
4	9	14	19	24	29
5	10	15	20	25	30

We may use any of the above lists! We will use the last one given above to continue our example on the next page. Then, the procedures may be practiced on the other lists provided on this page if desired.

As mentioned above, once we decide that we want to find all of the prime numbers between 1 and 30, we write all of the integers between 1 and 30.

1	6	11	16	21	26
2	7	12	17	22	27
3	8	13	18	23	28
4	9	14	19	24	29
5	10	15	20	25	30

Next, we follow the steps given below.

Steps to follow:

1. Cross out 1. The number 1 is not prime. Circle 2. The number 2 is the smallest prime number.
2. Cross out all multiples of 2. These numbers cannot be prime. Why?
3. Circle 3. It must be prime because it was not crossed out as a multiple of 2.
4. Cross out all multiples of 3. These numbers cannot be prime. Why?
5. Circle the next smallest number that has not been crossed out. It must be prime! Why?
6. Cross out all multiples of the number you have just circled. They cannot be prime! Why not?
7. Repeat steps 5 and 6 until every number from 1 to 30 is either circled or crossed out.

Hint for advanced thinking! If we pay attention to the numbers that are freshly crossed out by a given number, we find that the number 2 crosses out the number 4 first. The number 3 is the first number to cross out the number 9. The number 5 is the first number to cross out the number 25. The number 7 is the first number to cross out the number 49. Any number smaller than 49 was already crossed out by a prime number smaller than 7. Since $5^2 = 25$, $7^2 = 49$, and $25 < 30 < 49$, it turns out that once we circle 7 and 7 does not cross out anything new, then we can circle all remaining numbers! This means that in a very few steps, we are done! (We have identified all of the prime numbers between 1 and 30.) This concept is harder to get used to, and it is exciting once it makes sense.

Using the Sieve of Eratosthenes to Find the Prime Numbers Between 1 and 100

Once we decide that we want to find all of the prime numbers between 1 and 100, we write all of the integers between 1 and 100 in order from least to greatest. Let us use the list provided below and work down each column beginning with 1 and ending with 100, in order. We will use the steps provided below the list.

1	21	41	61	81
2	22	42	62	82
3	23	43	63	83
4	24	44	64	84
5	25	45	65	85
6	26	46	66	86
7	27	47	67	87
8	28	48	68	88
9	29	49	69	89
10	30	50	70	90
11	31	51	71	91
12	32	52	72	92
13	33	53	73	93
14	34	54	74	94
15	35	55	75	95
16	36	56	76	96
17	37	57	77	97
18	38	58	78	98
19	39	59	79	99
20	40	60	80	100

Steps to follow:

1. Cross out 1. The number 1 is not prime. Circle 2. The number 2 is the smallest prime number.
2. Cross out all multiples of 2. These numbers cannot be prime. (Why?)
3. Circle 3. It must be prime because it was not crossed out as a multiple of 2.
4. Cross out all multiples of 3. These numbers cannot be prime. (Why?)
5. Circle the next smallest number that has not been crossed out. It must be prime! (Why?)
6. Cross out all multiples of the number you have just circled. They cannot be prime! (Why not?)
7. Repeat steps 5 and 6 until every number from 1 to 100 is either circled or crossed out.

Reflections on the Sieve of Eratosthenes

1. We used the Sieve of Eratosthenes to identify all of the prime numbers between 1 and 30, then to identify all of the prime numbers between 1 and 100. Is it possible to use the Sieve of Eratosthenes to identify all of the prime numbers between 1 and any integer n, where n is greater than 1? YES! We can just write down all of the integers between 1 and n. Then, we do exactly same steps all the way through the list until all the numbers are either circled (prime numbers) or crossed out (the number 1 and the composite numbers).

2. What is a sieve? Look it up in a dictionary!

3. Who was Eratosthenes and when did he live?

4. According to the advanced thinking hint of the example for finding the prime numbers between 1 and 30 using the Sieve of Eratosthenes, the number 7 is the last prime number that really requires us to cross out a never-before crossed-out number smaller than 100. Why? Once 7 is circled, we cross out all multiples of 7. Well, $14 = 7 \cdot 2$ and all the multiples of 2 were crossed out! $21 = 7 \cdot 3$, and all the multiples of 3 were crossed out. According to the same logic, 28, 35, and 42 were already crossed out as multiples of 2, 3, and 5 in prior steps. The number $49 = 7 \cdot 7$ is the first number that has not been crossed out! There will be another new number: $77 = 7 \cdot 11$. Once all the multiples of 7 are crossed out, 11 is the next prime number we circle. Well, 22, 33, 44, 55, 66, 77, 88, and 99 have already been crossed out! If 110 were on the list, it would have been crossed out as a multiple of 2. The number $121 = 11 \cdot 11$ is the first number that the prime number 11 would have caused us to cross out! And, we know that 121 is greater than 100.

   We have just established that $7^2 = 7 \cdot 7 = 49$ and $49 < 100$, while $11^2 = 11 \cdot 11 = 121$ and $121 > 100$. Because of this, all the prime numbers greater than or equal 11 would *not* cause us to cross out a new number less than 100. That means that we can just circle 11 and all the *not yet crossed-out numbers* that are greater than 11. They are all the remaining prime numbers between 1 and 100.

   If we want to identify all the prime numbers between 1 and a given integer n, where n is greater than 1, we can save time if we think about numbers and their squares. Each time we circle a prime number, we may ask what the square of the number is that we just circled. If the square of the prime number we just circled is greater than n, then we may circle all remaining numbers! Or we can look for perfect squares and find the two numbers whose squares are just below and just above n.

5. If the discussion in 4. above seems too complicated to think about at first, we do not have to worry about it at all in order to get the work done. We can just follow the steps as written. If the number we use as our greatest number in the group is very large, then the ideas can save us a lot of time. For that reason, some additional explanation and practice is provided. It will be optional depending on interest and time!

6. Following the reflections, the work is shown step-by-step for using the Sieve of Eratosthenes for $n = 30$. Finally, the result for $n = 100$ is shown. The prime numbers are circled. The number 1 and the composite numbers are crossed out.

7. It is fun to use the Sieve of Eratosthenes to find all the prime numbers between 1 and an integer n, where n is greater than 1. For practice, use the Sieve of Eratosthenes to find all the prime numbers between the following numbers.

   (a) 1 and 50
   (b) 1 and 200
   (c) 1 and 75
   (d) 1 and 150
   (e) 1 and 300

8. What is the greatest prime number that will require us to cross out a new number less than n given in each pair below? (Each pair below has the form 1 and n.)

   (a) 1 and 50

   [The answer is 7 since $7^2 = 49$ and $49 < 50$. The number 11 is the next number we will circle, $11^2 = 121$, and $121 > 50$.]

   (b) 1 and 200

   [The answer is 13 since $13^2 = 169$ and $169 < 200$. The multiples of 14 have already been crossed out since they are all multiples of 2. The multiples of 15 have already been crossed out since they are all multiples of 3. The multiples of 16 have already been crossed out since they are multiples of 2. Also, $17^2 > 200$.]

   (c) 1 and 75

   [The answer is 7 since $7^2 = 49 < 75$ and $11^2 = 121 > 75$.]

   (d) 1 and 150

   [The answer is 11 since $11^2 = 121 < 150$ and $13^2 = 169 > 150$.]

   (e) 1 and 300

   [The answer is 17 since $17^2 < 300$. The number 18 would already have been crossed out as a composite number, but $19^2 = 361 > 300$.]

Using the Sieve of Eratosthenes to Find the Prime Numbers Between 1 and 30:

Step-by-Step Results

Step 1: Cross out 1. The number 1 is not prime. Circle 2. The number 2 is the smallest prime number.

~~1~~	6	11	16	21	26
2	7	12	17	22	27
3	8	13	18	23	28
4	9	14	19	24	29
5	10	15	20	25	30

Step 2: Cross out all multiples of 2. These numbers cannot be prime! Why?

~~1~~	~~6~~	11	~~16~~	21	~~26~~
2	7	~~12~~	17	~~22~~	27
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	9	~~14~~	19	~~24~~	29
5	~~10~~	15	~~20~~	25	~~30~~

Step 3: Circle 3. It must be prime because it was not crossed out as a multiple of 2.

~~1~~	~~6~~	11	~~16~~	21	~~26~~
2	7	~~12~~	17	~~22~~	27
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	9	~~14~~	19	~~24~~	29
5	~~10~~	15	~~20~~	25	~~30~~

Step 4: Cross out all multiples of 3. These numbers cannot be prime! Why?

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	25	~~30~~

Step 5: Circle the next smallest number that has not been crossed out. (So, we circle the number 5.) It must be prime! (Why?)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	25	~~30~~

Step 6: Cross out all multiples of the number you have just circled. They cannot be prime! (Why not?)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

Step 7: Repeat Steps 5 and 6 until every number between 1 and 30 is either circled or crossed out. Circle the next smallest number that has not been crossed out. (So, we circle 7. This is Step 5 repeated.)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

Step 7: Repeat Steps 5 and 6 until every number between 1 and 30 is either circled or crossed out. Cross out all multiples of the number you have just circled. (So, we now cross out all multiples of 7. This is Step 6 repeated.)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

Step 7: Repeat Steps 5 and 6 until every number between 1 and 30 is either circled or crossed out. Circle the next smallest number that has not been crossed out. (So, we circle 11. This is Step 5 repeated.)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

Step 7: Repeat Steps 5 and 6 until every number between 1 and 30 is either circled or crossed out. Cross out all multiples of the number you have just circled. (So, we now cross out all multiples of 11. This is Step 6 repeated.)

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

Now we can circle the remaining numbers that have not been crossed out.

~~1~~	~~6~~	11	~~16~~	~~21~~	~~26~~
2	7	~~12~~	17	~~22~~	~~27~~
3	~~8~~	13	~~18~~	23	~~28~~
~~4~~	~~9~~	~~14~~	19	~~24~~	29
5	~~10~~	~~15~~	~~20~~	~~25~~	~~30~~

The prime numbers between 1 and 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. So, there are 10 prime numbers between 1 and 30.

The Prime Numbers Between 1 and 100

Finally, the result for $n = 100$ is shown. Many thanks to Eratosthenes!

~~1~~	~~21~~	41	61	~~81~~
2	~~22~~	~~42~~	~~62~~	~~82~~
3	23	43	~~63~~	83
~~4~~	~~24~~	~~44~~	~~64~~	~~84~~
5	~~25~~	~~45~~	~~65~~	~~85~~
~~6~~	~~26~~	~~46~~	~~66~~	~~86~~
7	~~27~~	47	67	~~87~~
~~8~~	~~28~~	~~48~~	~~68~~	~~88~~
~~9~~	29	~~49~~	~~69~~	89
~~10~~	~~30~~	~~50~~	~~70~~	~~90~~
11	31	~~51~~	71	~~91~~
~~12~~	~~32~~	~~52~~	~~72~~	~~92~~
13	~~33~~	53	73	~~93~~
~~14~~	~~34~~	~~54~~	~~74~~	~~94~~
~~15~~	~~35~~	~~55~~	~~75~~	~~95~~
~~16~~	~~36~~	~~56~~	~~76~~	~~96~~
17	37	~~57~~	~~77~~	97
~~18~~	~~38~~	~~58~~	~~78~~	~~98~~
19	~~39~~	59	79	~~99~~
~~20~~	~~40~~	~~60~~	~~80~~	~~100~~

SI SESSION PLAN

Session Date: _____ Leader: _____

Course Instructor: _____ Course Name: Math B50, B60, B70 or B22 CRN:_____

Objective: What **one goal** does this group **most** need to accomplish by the end of this session?
The group will practice mathematical skills and vocabulary by finding sums, differences, products, and quotients as they play many short, two-player games with dice and compute, analyze, and discuss the results mathematically as a group.

Before you start, don't forget:

1. Has everyone logged in or signed an attendance sheet?
2. Did you write a brief agenda for the session on the white board?

Content to cover:	**How you will cover it:** (see ideas on back)
Warm-up—	Welcome.
Welcome.	Students pair up and assign "odd" and "even" players.
Organize in pairs and pass out material.	Pass out materials. Discuss rules and how to keep score
	with tally marks. SI Leader will keep the clock.
	Please see attached handout. Play Games 1 – 4 as
Work-out-- Play Game 1 three times.	outlined in the handout. During the group discussion,
Rules Change	give the group plenty of time to think and perhaps argue!
Game 2	For classes such as Math B50, B60, and B70, cover Part I.
Rules Change	For classes such as Math B22, cover Part I and Part II.
Game 3	If there is time or a second SI Session is done, Math B50,
Rules Change Names "Simplified" and "Not Simplified"	B60, and B70 students can certainly cover Part II.
Game 4	
	SI Leader: Large Group Discussion. Students can write on
Discuss the games as a group. Proceed to Part I.	board. During the discussion, the SI Leader can introduce
Depending on time and interest, proceed to Part II.	topics such as fairness as an important concept in math.
(For example, Part II will most likely be incorporated in	Government, politics, business, and international
Math B22 since probability theory is discussed in this course.)	relations all depend on rules, games, and issues of
Cool-down-- Review content that was practiced.	fairness.
Vocabulary & Skills, Identify New Thoughts	Game Theory.

See backside ⟹

AFTER YOUR SESSION IS FINISHED Number of students who came: _____

1. Give your session an overall grade: A B C D F
2. Explain why you gave your session the grade you just circled. What went well? What didn't go so well? What can you do differently for the next session?

3. Attach copies of any handouts you used.

Possible Ways to Cover Content

These are just several ideas. There are many more, and feel free to develop your own methods. Basically, if a technique promotes learning by engaging students, then it's a great strategy!

Warm-up Ideas:

Write opening question/statement on board and invite discussion. For example:
(General Question) How are you feeling today?
(Habits of Mind) What does it mean to PERSIST?
How are you guys doing at staying ORGANIZED?
Why do you think students do better at BC when they get INVOLVED?
How will your learning at BC benefit you for a lifetime?
(Content Questions) See if you can work out this review problem from last session.
What is the main issue that you are having difficulty with in class?
Please describe the steps of the _____ process.

(A Quote You Like) What does this mean to you?
"Success is the sum of small efforts, repeated day in and day out."

Work-out Ideas: Have them interact with the course content by using study strategies.

Quiz Questions: SI Leader quizzes group at random; students pair up and quiz each other
Working with Notes: Show them how to rewrite their notes by paraphrasing them, how to make study guides, how to create flash cards, how to recite information pacing back and forth
Using Whiteboard: Have them work out problems, make lists, model their ideas, etc. on the whiteboard
Games: Try bingo, jeopardy, etc. Many ideas can be found on the Internet :=).

Cool-down Ideas: Don't forget to end your session on a strong note!

Ask them to answer orally or on a piece of scratch paper to turn in to you:
Predict the next lecture: What do you think the next lecture in the class will be about?
Summarize session: Write a one-sentence summary of our session today.
Muddiest Point: Write down one point that is still unclear to you.

Policy: Turn in 1 completely filled out Session Plan per week every Monday by 5 PM to avoid a Missing Paperwork warning.

Mathematical Practice with Two-Player Dice Games

Materials Needed

- two dice
- paper and pencil to keep score
- Timer or clock to keep time
- Prior Knowledge: Odd Integers, Even Integers, Simplification of Fractions, Equivalent Representations of the Number 1 ($1/1 = 1$, $2/2 = 1$, ...).

Game 1: SUM

Rules

1. Choose one person to be "odd" and the other to be "even."
2. Roll the dice. Each person rolls one die.
3. Calculate the sum of the two numbers on the dice.
4. If the sum is an odd number, the "odd" player scores a point. If the sum is an even number, the "even" player scores a point. (Remember that zero is an even number.)
5. Keep score for both players as tally marks with pencil and paper.
6. Roll the dice for 2 minutes.
7. The winner of the game is the person with the most points.
8. Play the game three times in a row (6 minutes total) and record the number of times each person wins.

Game 2: DIFFERENCE

Rules

1. Choose one person to be "odd" and the other to be "even."
2. Roll the dice. Each person rolls one die.
3. Calculate the difference of the two numbers. (We can find the difference as the larger number minus the smaller number. If the numbers are equal, then the difference is zero.)
4. If the difference is an odd number, the "odd" player scores a point. If the sum is an even number, the "even" player scores a point. (Remember that zero is an even number.)
5. Keep score for both players as tally marks with pencil and paper.
6. Roll the dice for 2 minutes.
7. The winner of the game is the person with the most points.
8. Play the game three times in a row (6 minutes total) and record the number of times each person wins.

Game 3: PRODUCT

Rules

1. Choose one person to be "odd" and the other to be "even."
2. Roll the dice. Each person rolls one die.
3. Calculate the product of the two numbers.
4. If the product is an odd number, the "odd" player scores a point. If the product is an even number, the "even" player scores a point. (Remember that zero is an even number.)
5. Keep score for both players as tally marks with pencil and paper.
6. Roll the dice for 2 minutes.
7. The winner of the game is the person with the most points.
8. Play the game three times in a row (6 minutes total) and record the number of times each person wins.

Game 4: QUOTIENT

Rules

1. Choose one person to be "simplified" and the other to be "not simplified."
2. Roll the dice. Each person rolls one die.
3. Calculate the quotient of the two numbers by placing the smaller number in the numerator and the larger number in the denominator. If the two numbers are equal, then the simplified version will be 1. Count this result as a point for the "not simplified" player.
4. If the quotient is a simplified fraction, the "simplified" player scores a point. If the quotient is not yet in simplified form (e.g. 2/6) then the "not simplified" player scores a point.
5. Keep score for both players as tally marks with pencil and paper.
6. Roll the dice for 2 minutes.
7. The winner of the game is the person with the most points.
8. Play the game three times in a row (6 minutes total) and record the number of times each person wins.

Note: The inspiration for these games came from the National Council of Teachers of Mathematics (NCTM) 1988 Yearbook for K-12 Teachers. Games 2, 3, and 4 were included in that edition of the NCTM Yearbook articles "Find the Fairer Game" and "Diet Fractions."

Group Discussion After Playing Each of Games 1 - 4 Three Times

Part I

1. Compile the winnings for all the "odd" and "even" players for Games 1, 2, and 3. Compile the winnings for "simplified" and "not simplified" players in Game 4. (Depending on time and desire for simplicity, the compilation could be achieved by totaling the outcome of the three games for each pair of players or by totaling the number of tally marks for the whole class for all the minutes played.)

2. Discuss whether or not the game seems "fair." For fairness, we would expect that the two players in the game win approximately half the time. If a game is not fair, we will see an imbalance in the outcomes of "win" and "lose" for the two players with only one of the players dominating by winning more often.

   (a) Does Game 1 appear to be fair? YES or NO
   (b) Does Game 2 appear to be fair? YES or NO
   (c) Does Game 3 appear to be fair? YES or NO
   (d) Does Game 4 appear to be fair? YES or NO

3. Is it possible to do a ranking of the four games from "Most Fair Game" to "Least Fair Game?"

Part II Mathematical Analysis of the Fairness of Each Game. In this section, a fair two-player game is one in which the probability that each player wins is exactly 1/2. While the discussions in Part I may be held with any level of mathematics student, the following questions are for students who are studying probability or statistics to tackle.

1. Game 1: Analysis of Fairness

   (a) Find the probability that "odd" wins in Game 1.
   (b) Find the probability that "even" wins in Game 1.
   (c) Is the game fair or unfair?

2. Game 2: Analysis of Fairness

   (a) Find the probability that "odd" wins in Game 2.
   (b) Find the probability that "even" wins in Game 2.
   (c) Is the game fair or unfair?

3. Game 3: Analysis of Fairness

   (a) Find the probability that "odd" wins in Game 3.
   (b) Find the probability that "even" wins in Game 3.
   (c) Is the game fair or unfair?

4. Game 4: Analysis of Fairness

   (a) Find the probability that "simplified" wins in Game 4.
   (b) Find the probability that "not simplified" wins in Game 4.
   (c) Is the game fair or unfair?

5. Now rank the four games in terms of fairness from "most fair" to "least fair."

Help for Part II

If the students get stuck in Questions 1-4 in Part II, suggest that they examine all the possible combinations when two dice are used in the games. An example of such a tool in the analysis of who wins in Game 1 is the chart provided below. Die 1 has outcomes 1, 2, 3, 4, 5, and 6. These are located in the first column. Die 2 has outcomes 1, 2, 3, 4, 5, and 6. These are located in the first row. Then, the sum of the two numbers could be written in the body of the table. Afterwards, "odd" or "even" of these sums would be determined. In the table below, only the final result of who wins is shown. For example, $1 + 1 = 2$, and 2 is even. Once such a chart or table is made for each of the games, the probabilities requested are easier to calculate, and Question 5 is much easier to do because P(either player wins) = 1/2 is the definition of "fair play."

Game 1: Which Player Wins Each Outcome of the Two Dice?

There are 36 equally likely outcomes. In exactly 18 of the 36, "odd" wins. In exactly 18 of the 36, "even" wins. P(odd wins) = 18/36 = 1/2, and P(even wins) = 18/36 = 1/2. Game 1 is a fair game.

Game 1	1	2	3	4	5	6
1	even	odd	even	odd	even	odd
2	odd	even	odd	even	odd	even
3	even	odd	even	odd	even	odd
4	odd	even	odd	even	odd	even
5	even	odd	even	odd	even	odd
6	odd	even	odd	even	odd	even